九十六學年第二學期 PHYS2320 電磁學 第二次期中考(共兩頁)

[Griffiths Ch. 9]　2008/05/13,　10:10am–12:00am, 教師：張存績

記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

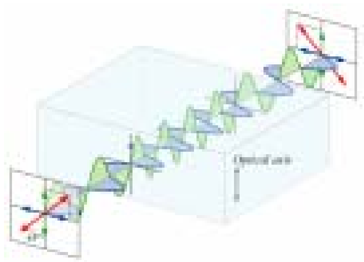
1. (10%, 10%) The generalized dielectric constant.

Assume there are N molecules per unit volume and Z electrons per molecule. Each molecule contains f_j electrons with binding frequency ω_j and damping factor γ_j, where $Z=\Sigma f_j$. The electrons could be classified into two groups, bound electrons and free electrons. The free electrons are denoted as f_0, $\omega_0=0$, and γ_0.

(a) Find the general polarization **P**.

(b) Find the general form of the complex dielectric constant.

2. (10%, 10%) Birefringence: the wave plate



Linearly polarized light entering a wave plate can be resolved into two waves, parallel and perpendicular to the optical axis of the wave plate. In the plate, assume that the parallel wave (k_y) propagates slightly slower than the perpendicular one (k_x).

Near side: $f_0 = A_0\cos(k_x z_0 - \omega t)\hat{\mathbf{x}} + A_0\cos(k_y z_0 - \omega t)\hat{\mathbf{y}}$

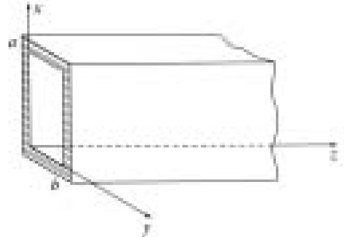
Far side: $f_d = A_0\cos(k_x(z_0+d) - \omega t)\hat{\mathbf{x}} + A_0\cos(k_y(z_0+d) - \omega t)\hat{\mathbf{y}}$

(a) At the far side of the plate, can we change the polarization of the the resulting combination orthogonal to its entrance state? At what condition?

(b) Is it possible to form a right or left hand circular polarization? At what condition?

3. (10%, 10%) A wave is propagating in a rectangular waveguide with fundamental TE_{10} mode.

$B_z(x,z,t) = B_0\cos(\pi x/a)\cos(kz - \omega t)$.



(a) Find E_x, E_y, B_x, and B_y?

(b) Find the surface current **K** on the bottom of the inner wall (the yz plane)?

4. (10%, 10%) A coaxial transmission line of inner radius *a* and outer radius *b* is filled with a dielectric material of permittivity $2\varepsilon_0$ and permeability μ_0.

(a) Find **E** and **B**.

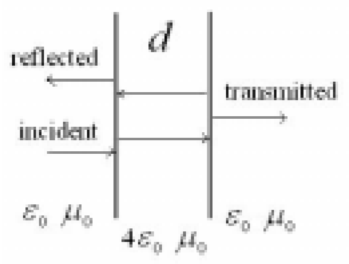
(b) Find the electric u_e and magnetic u_m field density. Are they the same?

[Hint 1: Reduce the problem to two dimensions.]

[Hint 2: $\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$,

$$\nabla \times \mathbf{v} = (\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z})\hat{\mathbf{s}} + (\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s})\hat{\boldsymbol{\varphi}} + \frac{1}{s}(\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi})\hat{\mathbf{z}}$$

5. (6%, 7%, 7%) A plane wave is normally incident onto a lossless dielectric slab of thickness *d* and dielectric constant $4\varepsilon_0$, as shown in the figure.



Incident wave: $\begin{cases} \mathbf{E}_I(z,t) = E_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \mathbf{B}_I(z,t) = \frac{1}{v_1} E_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{cases}$

(a) Write down the reflected wave, transmitted wave, and wave in the dielectric slab.

(b) Determine the reflection coefficient *R* and the transmission coefficient *T* as a function of propagating constant *k* and thickness *d*.

(c) If the frequency of the incident wave is 30 GHz, what is the minimum thickness *d* such that no reflection is observed?

[Hint: Assume the media is linear and there is no free charge and no free current at the interface.]

(a)

Let $\mathbf{x}(t)=\mathbf{x}_0 e^{-i\omega t}$ and substitute

$$\begin{cases}\mathbf{x}(t)=\mathbf{x}_0 e^{-i\omega t}\\ \mathbf{E}(\mathbf{x},t)=\mathbf{E}(0)e^{-i\omega t}\end{cases} \quad \text{into} \quad m(\ddot{\mathbf{x}}+\gamma\dot{\mathbf{x}}+\omega_0^2\mathbf{x})=-e\mathbf{E}(\mathbf{x},t),$$

we obtain $m(-\omega^2-i\omega\gamma+\omega_0^2)\mathbf{x}_0=-e\mathbf{E}(0)$ with the solution:

$$\mathbf{x}_0=-\frac{e}{m}\frac{\mathbf{E}(0)}{\omega_0^2-\omega^2-i\omega\gamma} \quad \Rightarrow \quad \mathbf{x}(t)=-\frac{e}{m}\frac{\mathbf{E}(0)e^{-i\omega t}}{\omega_0^2-\omega^2-i\omega\gamma}$$

Divide the electrons in the medium into $\begin{cases}\text{bound electrons: } \omega_j\neq 0\\ \text{free electrons: } \omega_j=0,\ f_j=f_0,\ \gamma_j=\gamma_0\end{cases}$

$$\mathbf{P}(\mathbf{x})=N\sum_j f_j\mathbf{p}_j=\left(\frac{Ne^2}{m}\sum_{\text{bound}}\frac{f_j}{\omega_j^2-\omega^2-i\omega\gamma_j}+i\frac{Ne^2}{m}\frac{f_0}{-i\omega^2+\omega\gamma_0}\right)\mathbf{E}(\mathbf{x})=\varepsilon_0\chi_e\mathbf{E}(\mathbf{x})$$

(b)

$$\begin{cases}\mathbf{D}(\mathbf{x})\equiv\varepsilon_0\mathbf{E}(\mathbf{x})+\mathbf{P}(\mathbf{x})=\varepsilon\mathbf{E}(\mathbf{x})\\ \varepsilon=\varepsilon_0(1+\chi_e)\end{cases}$$

to fields with $\exp(-i\omega t)$ dependence, we obtain $\mathbf{D}(\mathbf{x})=\varepsilon\mathbf{E}(\mathbf{x})$

$$\text{with } \varepsilon=\underbrace{\varepsilon_0+\frac{Ne^2}{m}\sum_{j\ (\text{bound})}\frac{f_j}{\omega_j^2-\omega^2-i\omega\gamma_j}}_{\varepsilon_b}+i\underbrace{\frac{Ne^2 f_0}{m\omega(\gamma_0-i\omega)}}_{\sigma/\omega}=\varepsilon_b+i\frac{\sigma}{\omega}$$

2.

(a)Yes, we can.

Near side: z_0=0 $\quad f_0=A_0\cos(-\omega t)\hat{\mathbf{x}}+A_0\cos(-\omega t)\hat{\mathbf{y}}$.

The field is polarized in the first and third quadrants.

Far side: $\quad f_d=A_0\cos(k_y d+n\pi-\omega t)\hat{\mathbf{x}}+A_0\cos(k_y d-\omega t)\hat{\mathbf{y}}=-A_0\cos(k_y d-\omega t)\hat{\mathbf{x}}+A_0\cos(k_y d-\omega t)\hat{\mathbf{y}}$,

where $(k_x-k_y)d=n\pi,\ n=1,3,5,\ldots$

The resulting field is linearly polarized in the second and fourth quadrants.

(b) Yes, we can.

Near side: z_0=0 $\quad f_0=A_0\cos(-\omega t)\hat{\mathbf{x}}+A_0\cos(-\omega t)\hat{\mathbf{y}}$.

The field is polarized in the first and third quadrant.

Far side: $\quad f_d=A_0\cos(k_y d+\frac{n}{2}\pi-\omega t)\hat{\mathbf{x}}+A_0\cos(k_y d-\omega t)\hat{\mathbf{y}}=-A_0\sin(k_y d-\omega t)\hat{\mathbf{x}}+A_0\cos(k_y d-\omega t)\hat{\mathbf{y}}$,

where $(k_x-k_y)d=\frac{n}{2}\pi,\ n=1,5,9\ldots$ LHCP

Far side: $\quad f_d=A_0\cos(k_y d+\frac{n}{2}\pi-\omega t)\hat{\mathbf{x}}+A_0\cos(k_y d-\omega t)\hat{\mathbf{y}}=A_0\sin(k_y d-\omega t)\hat{\mathbf{x}}+A_0\cos(k_y d-\omega t)\hat{\mathbf{y}}$,

where $(k_x-k_y)d=\frac{n}{2}\pi,\ n=3,7,11\ldots$ RHCP

3.

$$E_x=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial E_z}{\partial x}+\omega\frac{\partial B_z}{\partial y}\right),\quad E_y=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial E_z}{\partial y}-\omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial B_z}{\partial x}-\frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right),\quad B_y=\frac{i}{(\omega/c)^2-k^2}\left(k\frac{\partial B_z}{\partial y}+\frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

(a) $B_z(x,z,t) = B_0 \cos(\pi x/a)\cos(kz-\omega t), E_z = 0$

$$E_x = \frac{i}{(\omega/c)^2 - k^2}(\omega\frac{\partial B_z}{\partial y}) = 0, \quad E_y = \frac{i}{(\omega/c)^2 - k^2}(-\omega\frac{\partial B_z}{\partial x}) = \frac{i}{(\omega/c)^2 - k^2}\frac{\omega\pi}{a}\sin(\frac{\pi}{a}x)\cos(kz-\omega t)$$

$$B_y = \frac{i}{(\omega/c)^2 - k^2}(k\frac{\partial B_z}{\partial y}) = 0, \quad B_x = \frac{i}{(\omega/c)^2 - k^2}(k\frac{\partial B_z}{\partial x}) = \frac{i}{(\omega/c)^2 - k^2}(-\frac{k\pi}{a})\sin(\frac{\pi}{a}x)\cos(kz-\omega t)$$

(b) The surface current **K** on the bottom of the inner wall.

At x=0, the only non-vanished term is $B_z(x=0) = B_0\cos(kz-\omega t)$.

$$\mathbf{K} = \hat{\mathbf{n}}\times\mathbf{H} = \hat{\mathbf{x}}\times B_0\cos(kz-\omega t)\hat{\mathbf{z}} = -B_0\cos(kz-\omega t)\hat{\mathbf{y}}$$

4.

(a) The problem is reduced to two dimensions.

$$\left.\begin{array}{l}\nabla_t\times\mathbf{E}=0\\ \nabla_t\times\mathbf{B}=0\end{array}\right\}\Rightarrow\begin{cases}\mathbf{E}=-\nabla_t\phi_E,\ \nabla_t^2\phi_E=0 & \text{electrostatic}\\ \mathbf{B}=-\nabla_t\phi_B,\ \nabla_t^2\phi_B=0 & \text{magnetostatic}\end{cases}$$

Electrostatic: the infinite line charge: $\mathbf{E}_0(s,\phi,z,t) = \dfrac{A\cos(kz-\omega t)}{s}\hat{\mathbf{s}}$

Magnetostatic: an infinite straight current: $\mathbf{B}(s,\phi,z,t) = \dfrac{B\cos(kz-\omega t)}{s}\hat{\boldsymbol{\varphi}}$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \ \rightarrow\ \left.\begin{array}{l}\dfrac{\partial E_\phi}{\partial z} = \dfrac{kA}{s}\sin(kz-\omega t)\\ -\dfrac{\partial B_s}{\partial t} = \dfrac{\omega B}{s}\sin(kz-\omega t)\end{array}\right\}\ B = \frac{k}{\omega}A$$

(b)

$$u_e = \frac{1}{2}(2\varepsilon_0)E^2 = \varepsilon_0\frac{A^2\cos^2(kz-\omega t)}{s^2}$$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2\mu_0}\frac{B^2\cos^2(kz-\omega t)}{s^2} = \varepsilon_0\frac{A^2\cos^2(kz-\omega t)}{c^2s^2}(\frac{1}{2\mu_0\varepsilon_0}\frac{\omega^2}{k^2})$$

since $\nabla^2\mathbf{B} - \varepsilon\mu\dfrac{\partial^2\mathbf{B}}{\partial t^2} = 0 \Rightarrow k^2 = 2\mu_0\varepsilon_0\omega^2 \Rightarrow u_e = u_m$

Yes.

5.

(a)

Incident wave: $\begin{cases}\mathbf{E}_I(z,t) = E_{0I}e^{i(k_1z-\omega t)}\hat{\mathbf{x}}\\ \mathbf{B}_I(z,t) = \dfrac{1}{v_1}E_{0I}e^{i(k_1z-\omega t)}\hat{\mathbf{y}}\end{cases}$

Reflected wave: $\begin{cases}\mathbf{E}_R(z,t) = E_{0R}e^{i(k_1z-\omega t)}\hat{\mathbf{x}}\\ \mathbf{B}_R(z,t) = -\dfrac{1}{v_1}E_{0R}e^{i(-k_1z-\omega t)}\hat{\mathbf{y}}\end{cases}$

Wave in the dielectric slab: $\begin{cases} \mathbf{E}_d(z,t) = E_{0c}e^{i(k_2 z-\omega t)}\hat{\mathbf{x}} + E_{0d}e^{i(-k_2 z-\omega t)}\hat{\mathbf{x}} \\ \mathbf{B}_d(z,t) = \dfrac{1}{v_2}E_{0c}e^{i(k_2 z-\omega t)}\hat{\mathbf{y}} - \dfrac{1}{v_2}E_{0d}e^{i(-k_2 z-\omega t)}\hat{\mathbf{y}} \end{cases}$

Transmitted wave: $\begin{cases} \mathbf{E}_T(z,t) = E_{0T}e^{i(k_1 z-\omega t)}\hat{\mathbf{x}} \\ \mathbf{B}_T(z,t) = \dfrac{1}{v_1}E_{0T}e^{i(k_1 z-\omega t)}\hat{\mathbf{y}} \end{cases}$

(b)

Boundary condition:

Tangential E at z=0, $E_{0I} + E_{0R} = E_{0c} + E_{0d}$

Tangential B at z=0, $\dfrac{1}{v_1}(E_{0I} - E_{0R}) = \dfrac{1}{v_2}(E_{0c} - E_{0d})$

Tangential E at z=d, $E_{0c}e^{ik_2 d} + E_{0d}e^{-ik_2 d} = E_{0T}e^{ik_1 d}$

Tangential B at z=d, $\dfrac{1}{v_2}(E_{0c}e^{ik_2 d} - E_{0d}e^{-ik_2 d}) = \dfrac{1}{v_1}E_{0T}e^{ik_1 d}$

$$T = \left|\frac{E_{0T}}{E_{0I}}\right|^2 = 1 + \frac{1}{4}\left(\frac{k_1^2 - k_2^2}{k_1 k_2}\right)^2 \sin^2(k_2 d)$$

$$R = \left|\frac{E_{0R}}{E_{0I}}\right|^2 = \frac{1}{4}\left(1 + \frac{1}{4}\left(\frac{k_1^2 - k_2^2}{k_1 k_2}\right)^2 \sin^2(k_2 d)\right)\left(\frac{k_2^2 - k_1^2}{k_1 k_2}\right)^2 \sin^2(k_2 d)$$

(c)

$$R = \frac{1}{4}\left(1 + \frac{1}{4}\left(\frac{k_1^2 - k_2^2}{k_1 k_2}\right)^2 \sin^2(k_2 d)\right)\left(\frac{k_2^2 - k_1^2}{k_1 k_2}\right)^2 \sin^2(k_2 d) = 0$$

$$\sin(k_2 d) = 0 \quad \Rightarrow \quad k_2 d = \pi, \quad k_2 = \frac{4\pi f}{c}$$

$$d = \frac{c}{4f} = 0.25 \text{ cm}$$